7. Three Dimensional Geometric and Modeling Transformations

Contents

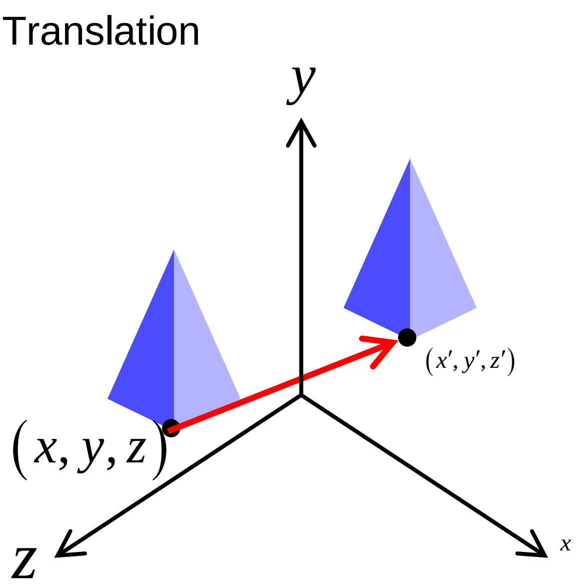
Introduction

- It is very similar to 2D. It uses 4x4 matrices rather than 3x3.
- A 3D point P is represented in homogeneous coordinates by a 4-dimensional vector

7.1 Translation

- A translation moves all points in an object along the same straight line path to new positions.
- The path is represented by a vector, called the translation or shift vector.

  We can write the components as



$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

7.2 Rotation

- For 3D object rotation transformation we need to pick an axis to rotate about and the amount of angular rotation.
- 3D rotation can be specified around any line in space

The most common choices are the X-axis, the Y-axis, and the Z-axis.

Rotation about Z-axis

Rotation about z axis in Three-dimensional

Z-axis rotation is identical to the 2D case:

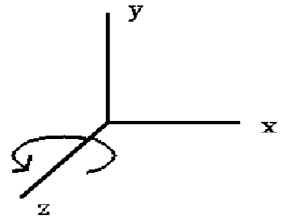
$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

The parameter θ is specifies the rotation angle. In homogeneous coordinate form ,the 3D z-axis rotation equations are expressed as

$$[x', y', z', 1] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



We can write more compactly as

$P' = R_z(\theta).P$

Transformation equations for rotation about the other two coordinate axes can be obtained with a cyclic permutation of the coordinate parameters x,y and z in the above equation. That is we use the replacements

$x \to y \to z \to x$

Rotation about X- axis

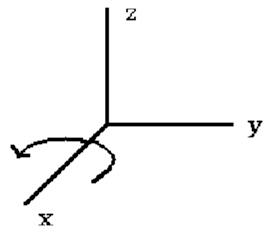
X-axis rotation looks like Z-axis rotation if replace: X axis with Y axis ,Y axis with Z axis
Z axis with X axis.So we do the same replacement in the equations:

Same argument as for rotation about *z* axis For rotation about *x* axis, *x* is unchanged

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

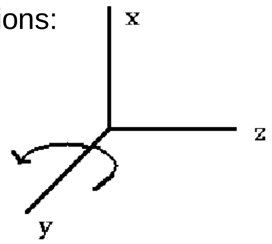


$$[x',y',z',1]=\begin{bmatrix}1&0&0&0\\0&\cos\theta&-\sin\theta&0\\0&\sin\theta&\cos\theta&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

$P' = R_x(\theta).P$

Rotation about y axis

- Y-axis rotation looks like Z-axis rotation if replace: X axis with Z axis ,Y axis with X axis ,Z axis with Y axis. So we do the same replacement in equations:
- Same argument as for rotation about *z* axis
- For rotation about *y* axis, *y* is unchanged



$x' = x\cos\theta + z\sin\theta$

$y' = y$

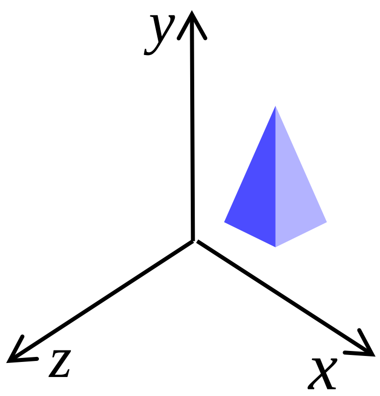
$z' = z\cos\theta - x\sin\theta$

$$[x',y',z',1]=\begin{bmatrix}\cos\theta&0&\sin\theta&0\\0&1&0&0\\-\sin\theta&0&\cos\theta&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

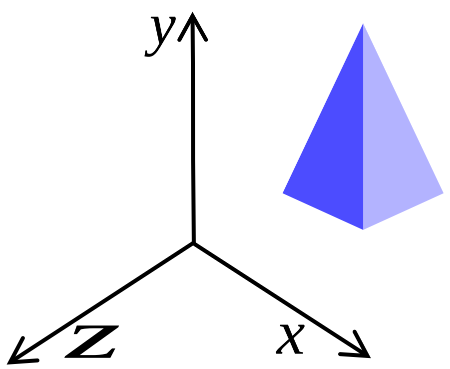
$P' = R_y(\theta).P$

7.3 Scaling

- Scaling changes the size of an object and involves the scale factors.
- The scaling parameters S_x , S_y and S_z are assigned any positive values.
- Scales the object about the origin, Here changes the size of the object along x,y and z-coordinate is same



The object before scaling



Object after scaling

Enlarging object also moves it from origin

$x' = S_x . x$

$y' = S_y . y$

$z' = S_z . z$

$$\mathbf{P}' = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \mathbf{S} \cdot \mathbf{P}$$

7.4 Reflections

- A Three-dimensional can be performed relative to a selected reflection axes or with respect to selected reflected plane.
- Three-dimensional reflection matrices are set up similar to those for two dimensional.
- Reflection relative to given axis are equivalent to 180° rotation about that axis.
- The reflection planes are either xy, xz or yz

- The matrix expression for the reflection transformation of a position P = (*x*, *y*, *z*) relative to *xy* plane can be written as:

$$RF_z = \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Transformation matrices for inverting *x* and *y* values are defined similarly, as reflections relative to *yz* plane and *xz* plane, respectively

7.5 Shears

The matrix expression for the shearing transformation of a position P = (*x*, *y*, *z*), to produce *z*-axis shear, can be written as:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- Parameters *a* and *b* can be assigned any real values. The effect of this transformation is to alter *x*- and *y*- coordinate values by an amount that is proportional to the *z* value, while leaving the *z* coordinate unchanged.
- Shearing transformations for the *x* axis and *y* axis are defined similarly.

7.6 Composite transformation

- We can form arbitrary affine transformation matrices by multiplying together rotation, translation, and scaling matrices
- Because the same transformation is applied to many vertices, the cost of forming a matrix M=ABCD is not significant compared to the cost of computing Mp for many vertices p
- The difficult part is how to form a desired transformation from the specifications in the application
- Consider the composite transformation matrix **M**=**ABC**
- When we calculate **Mp,** matrix C is the first applied, then B, then A
- Mathematically, the following are equivalent

  p' = **ABCp** = **A**(**B**(**Cp**))
- Hence composition order really matters.

Rotation About Point P

- Move fixed point P to origin
- Rotate by desired angle
- Move fixed point P back
- M = T(p_f) R(q) T(-p_f)